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3-TOTAL DIFFERENCE CORDIAL LABELING OF SOME GRAPHS

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ABSTRACT

We introduce a new graph labeling technique called k- total difference cordial labeling .Let $f:V(G)\to\{0,1,2,..,k-1\}$ be a map where $k \varepsilon N$ and $k>1$.For each edge uv assign the label $|f(u)-f(v)|$,f is called k-total difference cordial labeling of G if $|t_{df}(i)- t_{df}(j)|\leq 1$, $i,j\varepsilon\{0,1,,2,\ldots\ldots k-1\}$ where $t_{df}(x)$ denote the total number of vertices and the edges labeled with x. A Graph with a k- total difference cordial labeling is called k- total difference cordial graph. We investigate k-total difference cordial labeling of some graphs and study the 3-total difference cordial labeling behavior of Fan ,Sun flower,$T_n\Theta K_1$,$S(L_n)$,$S(K_{1,n})$,$S(P_n\Theta K_1)$.

Keywords: *Path, Cycle, Fan, Corona of graphs.*

I. INTRODUCTION

Consider All graph in this paper are finite, simple and undirected. Cahit[1] introduced the notion of cordial labeling of graphs.. Recently Ponraj[4 has been introduced the concept of k-total difference cordial graph . In [4] we prove that every graph is a subgraph of a connected k-total difference cordial graphs and investigate the k- total difference cordial labeling of some graph and 3- total difference cordial labeling behavior of path cycle, star, etc .Also we investigate the 4- total difference cordial labeling of several graphs in[5] . In this paper investigate 3-total difference cordial labeling of some graphs like Fan, Sun flower,$T_n\Theta K_1$,$S(L_n)$,$S(K_{1,n})$,$S(P_n\Theta K_1)$. Terms not in here from [2,3]

II. K-TOTAL DIFFERENCE CORDIAL LABELING

Definition 2.1

Let G be a graph and $f:V(G)\to\{0,1,2,\ldots\ldots,k-1\}$ be a map where $k \varepsilon N$ and $k>1$.For each edge uv assign the label $|f(u)-f(v)|$,f is called k-total difference cordial labeling of G if $|t_{df}(i)- t_{df}(j)|\leq 1$, $i,j\varepsilon\{0,1,2,\ldots\ldots k-1`\}$ where $t_{df}(x)$ denote the total number of vertices and the edges labeled with x. A Graph with admits a k- total difference cordial labeling is called k- total difference cordial graph.

III. PRELIMINARIES

Definition 3.1

The join of two graphs G_1 and G_2 is the graph $G_{1+} G_2$ with $V(G_{1+} G_2)=V(G_1)U(G_2)$,$E(G_{1+} G_2)= E(G_1)U E(G_1)U\{uv:u \varepsilon V(G_1),v \varepsilon V(G_2)$.

The graph $F_n = P_n + K_1$ is called the Fan.

Definition 3.2

The corona of G_1 with G_2,G_{12} is the graph obtained by taking one copy of G_2 and p_1 copies of G_2 and joining the i^{th} vertex of G_1 with an edge to every vertex in the i^{th} copy of $\mathbf{G_2}$.

Definition 3.3

The sunflower graph S_n is obtained from the wheel $W_n = C_n+K_1$ with central vertex v and the cycle C_n: $v_1v_2\ldots\ldots v_nv_1$ and new vertices $w_1w_2\ldots\ldots w_n$ where w_i is joined by vertices v_i,v_{i+1} (mod n).

Definition 3.4
Let P_n be the path $u_1 u_2 \ldots\ldots u_n$.The Triangular snake T_n is obtained from P_n with $V(T_n) = V(P_n) \cup \{ v_1v_2\ldots\ldots v_{n-1} \}$ and $E(T_n) = E(P_n) \cup \{ u_i v_i , u_{i+1} v_i : 1 \le i \le n-1\}$.

Definition 3.5.
An edge x = uv of G is said to be subdivided if it is replaced by the edges uw and wv where w is a vertex not in V (G).If every edge of G is subdivided ,the resulting graph is called the subdivision graph **S(G).**

Theorem 3.6[4].
If G is a (p, q) graph. Then G is a subgraph of c connected k-total different cordial graph.

Theorem 3.7 [4].
If $n \equiv 0 (\bmod\ k)$ then the star $K_{1,n}$ is k-total difference cordial.

Theorem 3.8[4].
The path P_n is 3-total difference cordial iff $n \neq 2$

Theorem 3.9[4].
The bistar $B_{n,n}$ is 3-total different cordial iff $n \equiv 1, 2 (\bmod 3)$.

Theorem 3.10[4].
The complete bipartite graph $K_{2,n}$ is 3-total difference cordial.

Theorem 3.11[4].
All combs are 3-total difference cordial.

Theorem 3.12[4].
All Wheels are 3-total difference cordial.

Theorem 3.13[4].
Helms H_n is 3-total difference cordial.

Theorem 3.14[4].
Any star is $K_{1,n}$ 3-total difference cordial.

Theorem 3.15[4].
AC_n is 3-Total difference cordial for all $n \geq 3$

Theorem 3.16[4].
$C_4(m)$ is 3-total difference cordial for all even values of m.

Theorem 3.17[4].
The subdivision of bistar$B_{n,n}$,$S(B_{n,n})$ is 3-total different cordial for all n.

Theorem 3.18[4].
$P_n \odot 2K_1$ is 3-total difference cordial for all n

IV. MAIN RESULTS

Theorem 4.1
The Fan F_n is 3-total difference cordial iff $n \geq 3$.

Proof:
Let $F_n = P_n + K_1$ and P_n be the path $u_1 u_2 \ldots u_n$, $V(K_1)=\{u\}$.

Case (1) n=1
$F_1 \approx P_2$, proof follows from [3.8].

Case (2) n=2
Suppose f is a 3-total difference cordial labeling of F_2 .Then $t_{df}(0) = t_{df}(1) = t_{df}(2) = 2$.Assume $f(u_1) = 2$.Clearly either $f(u_2) = 0$ or $f(u) = 0$.

Subcase (1) $f(u) = 0$ and $f(u_2) = 0$
$t_{df}(1)=0$,a contradiction

Subcase (2) $f(u) = 0$ and $f(u_2) = 1$
$t_{df}(1)=3$,a contradiction

Subcase (3) $f(u) = 0$ and $f(u_2) = 2$
$t_{df}(1)=0$,a contradiction

Subcase (4) $f(u) = 1$ and $f(u_2) = 0$
$t_{df}(1)=3$,a contradiction

Subcase (5) $f(u) = 1$, $f(u_1) = 2$ and $f(u_2) =$
, $t_{df}(1)=3$,a contradiction

Subcase (6) $f(u) = 1$, $f(u_1) = 2$ and $f(u_2) = 2$
$t_{df}(1)=3$,a contradiction

Case (3) n is odd and $n \geq 3$
Assign the label 0 to the vertex u,We now move to the path vertices. Assign the label 2 to the vertices $u_1 u_2 \ldots u_n$ and assign the label 1 to the vertices $u_{(n+3)/2}, u_{(n+5)/2}, \ldots, u_n$. Finally assign the label 0 to the vertex $u_{(n+1)/2}$

Case (4) n is even and $n > 2$
In this case assign the label 2 to the vertex u,We now move to the path vertices. Assign the label 2 to the vertices $u_1 u_2 \ldots u_{n/2}$ and assign the label 1 to the vertices $u_{(n+1)/2}, u_{(n+2)/2}, \ldots, u_{n-1}$. Finally assign the label 0 to the last path vertex u_n.
Since $t_{df}(0) = t_{df}(1) = t_{df}(2) = n$, f is a 3-total difference cordial labeling.

Theorem 4.2
If $n \equiv 0,1 \pmod 3$ then the sunflower graph is 3-total difference cordial.

Proof:
Assign the label 1 to the central vertex v.Next we assign the label 2 to the all rim vertices $v_1, v_2, \ldots, v_n$ of the wheel. We now consider $w_1, w_2, \ldots, w_n$.

Case 1: $n \equiv 0 \pmod 3$
Let $n = 3r, r \in N$.Assign the label 0 to the vertices $w_1, w_2, \ldots, w_r$ .Next assign the label 1 to the next vertices $w_{r+1}, w_{r+2}, \ldots, w_{2r}$.Finally assign the label 2 to the vertices $w_{2r+1}, w_{2r+2}, \ldots, w_{3r}$.

Case 2: $n \equiv 1 \pmod 3$

Let $n = 3r+1, r \varepsilon N$. In this case assign the label 0 to the vertices $w_1, w_2, \ldots, w_r, w_{r+1}$. Next assign the label 1 to the next vertices $w_{r+2}, w_{r+3}, \ldots, w_{2r}$. Finally assign the label 2 to the vertices $w_{2r+1}, w_{2r+2}, \ldots, w_{3r+1}$.
The table 1 shows that this labeling is 3-total difference cordial labels.

Table 1

Values of n	$t_{df}(0)$	$t_{df}(1)$	$t_{df}(2)$
$n \equiv 0 \pmod 3$	6r	6r	6r+1
$n \equiv 1 \pmod 3$	6r+3	6r+2	6r+2

Theorem 4.3
The corona of T_n with K_1, $T_n \Theta K_1$ is 3-total difference cordial for all even n.

Proof:
Let $V(T_n \Theta K_1) = V(T_n) \cup \{x_i : 1 \le i \le n-1\} \cup \{y_i : 1 \le i \le n\}$ and $E(T_n \Theta K_1) = E(T_n) \cup \{v_i x_i : 1 \le i \le n-1\} \cup \{u_i y_i : 1 \le i \le n\}$. Assign the label 2 to the all path vertices $u_1, u_2, \ldots, u_n$. Next assign the label 0 to the vertices $v_1, v_2, \ldots, v_{(n-2)/2}$. We now assign the label 2 to the vertices $v_{n/2}, v_{(n+2)/2}, \ldots, v_{n-1}$. Then assign the label 1 to the $x_1, x_2, \ldots, x_{n-2/2}$ vertices, assign the label 2 to the vertices $x_{n/2}, x_{(n+2)/2}, \ldots, x_{n-1}$. Finally assign the label 1 to the all $y_1, y_2, \ldots, y_n$ vertices. Since $t_{df}(0) = t_{df}(1) = t_{df}(2) = 9n-6/3$, f is a 3-total difference cordial labeling.

Theorem 4.4
$S(L_n)$ is a 3-total difference cordial labeling

Proof:
Let $V(L_n) = \{u_i, v_i \ (1 \le i \le n)\}$ and $E(L_n) = \{u_i v_i : (1 \le i \le n)\} \cup \{u_i u_{i+1}, v_i v_{i+1} \ (1 \le i \le n-1)\}$. Let x_i be the vertex which subdivided the edge $u_i v_i$ .and y_i, z_i be the vertex which subdivided the edges $u_i u_{i+1}$, $v_i v_{i+1}$ $(1 \le i \le n-1)$

Case (1) n is even $n > 2$
We assign the label 2 to the vertices $u_1, u_2, \ldots, u_n$ and assign the label 1 to the vertices $v_1, v_2, \ldots, v_n$. Next we assign the label 0 to the vertices $y_1, y_2, \ldots, y_{n/2}$ and $z_1, z_2, \ldots, z_{n/2}$ then assign the label 2 to the vertices $y_{(n+2)/2}, y_{(n+4)/2}, \ldots, y_n$ and $z_{(n+2)/2}, z_{(n+4)/2}, \ldots, z_n$. Next we assign the label 0 to the vertices $x_1, x_2, \ldots, x_n$.

Case (2) n is odd $n > 3$
Assign the label to the vertices $u_i, v_i \ (1 \le i \le n)$ as in case (1) .Next we assign the label 0 to the vertices $y_1, y_2, \ldots, y_{(n-1)/2}$. and $z_1, z_2, \ldots, z_{(n-1)/2}$ .Next we assign the label 2 to the vertices $y_{(n+1)/2}, y_{(n+3)/2}, \ldots, y_n$ and assign the label 1 to the vertices $z_{(n+1)/2}, z_{(n+3)/2}, \ldots, z_n$.

The Table 2 shows that this labeling is 3-total difference cordial labels

Table 2

Values of n	$t_{df}(0)$	$t_{df}(1)$	$t_{df}(2)$
n =0 (mod 3)	(11n-6)/3	(11n-6)/3	(11r-6)/3
n =1 (mod 3)	(11n-8)/6	(11n-5)/6	(11n-5)/6
n =2 (mod 3)	(11n+4)/3	(11n-4)/3	(11n-4)/3

Theorem 4.5
$S(K_{1,n})$ is 3-total difference cordial for all n.

Proof:
Let $V(S(K_{1,n})) = \{u, u_i, v_i : (1 \le i \le n)\}$ and $E(S(K_{1,n})) = \{uu_i, u_i v_i : (1 \le i \le n)\}$

Case (1) $n = 3r$

Assign the label 0 to the vertices u.Next assign the label 0 to the vertices $u_1,u_2,$ …... $,u_r$.we now assign the label 1 to the vertices $u_{r+1}, u_{r+2},$…..., u_{2r} .Then assign the label 2 to the vertices $u_{2r+1}, u_{2r+2},$…...., u_{3r}.

Subcase (1) When r is odd.

Assign the label 0 to the vertices $v_1,v_2,$….., $v_{(r-1)/2}$ .Next assign the label 1 to the vertices $v_{(r+1)/2},v_{(r+3)/2},$…...., v_r then assign the label 2 to the vertices $v_{r+1},v_{r+2},$…...., v_{3r} .

Subcase (2) When r is even.

Assign the label 0 to the vertices $v_1,v_2,$….. $,v_{r/2}$ .Next assign the label 1 to the vertices $v_{(r+2)/2},v_{(r+4)/2'}$…...., v_r then assign the label 2 to the vertices $v_{r+1},v_{r+2},$…...., v_{3r} .

Case (2) n=3r+1

As in case 1 assign the label to the vertices u,u_i,v_i $(1\leq i \leq n)$.Next assign the label 2,2 to the vertices u_{3r+1} and v_{3r+1}

Case (3) n=3r+2

As in case 1 and case 2 assign the label to the vertices u,u_i,v_i $(1\leq i \leq n-1)$.Next assign the label 1,1 to the vertices u_{3r+2} and v_{3r+2} .

The table 3 shows that the vertex labeling is a 3-total diference cordial labeling.

Table 3

Value of n	$t_{df}(0)$	$t_{df}(1)$	$t_{df}(2)$
3r	4r	4r+1	4r
3r+1	4r+2	4r+1	4r+2
3r+2	4r+3	4r+3	4r+3

Theorem 4.6

$S(P_n \odot K_1)$ is a 3-total difference cordial labeling

Proof:

Let P_n be the path $u_1 u_2$ …... u_n .Let $V(P_n \odot K_1) = V(P_n) \cup \{ v_i : 1\leq i \leq n \}$.Let x_i be the vertex subdivided the edge $u_i u_{i+1}$ $(1\leq i \leq n-1)$ .Let y_i be the vetex which subdivided $u_i v_i$ $(1\leq i \leq n)$.Since $S(P_n \odot K_1) \approx P_3$ and $S(P_2 \odot K_1) \approx P_7$ the proof follow from theorem

Case (1) $n = 3r$, $r \varepsilon N$, $r \geq 1$

Assign the label 2 to the vertices $u_1,u_2,$…..., u_n , $x_1, x_2,$…..., x_{n-1} .Next assign the label 2 to the vertices $y_1\, y_2$ …... y_{r+1} and $y_{r+2}\, y_{r+3}$ …... y_{2r}.Finally assign the label 1 to the remaining non-labelled vertices.

Case (2) $n = 3r + 1$, $r \varepsilon N$, $r \geq 1$

As in case (1) assign the label to the vertices u_i , v_i , y_i $(1\leq i \leq n-1)$ and x_i $(1\leq i \leq n-2)$ .Next assign the label 2,2 and 1 respectively to the vertices x_{n-1} , u_n, $y_n$and v_n.

Case (3) $n = 3r + 2$, $r \varepsilon N$, $r \geq 1$

Assign the label to the vertices u_i , v_i , y_i $(1\leq i \leq n-1)$ and x_i $(1\leq i \leq n-2)$ as in case (1) .Next assign the label 2,2,1 and 2 to the vertices x_{n-1} , u_n, $y_n$and v_n respectively.

The table 4 given below shows that this labeling is 3-total difference cordial labels.

Table 4

Value of n	$t_{df}(0)$	$t_{df}(1)$	$t_{df}(2)$
3r	8r-1	8r-1	8r-1
3r+1	8r+2	8r+2	8r+1
3r+2	8r+4	8r+5	8r+4

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REFERENCES

1. *I.Cahit,Cordial graphs:A weaker version of graceful and harmonious graphs,Ars Combinatoria,23(1987),201-207.*
2. *J.A.Gallian,A Dynamic survey of graph labeling,The Electronic Journal of Combinatorics,19 (2017) #Ds6.*
3. *F.Harary ,Graph theory, Addision Wesley New Delhi (1969).*
4. *R.Ponraj,S. Yesu Doss Philip and R.Kala ,k-total difference cordial of graphs (Submitted to the journal) .*
5. *R.Ponraj,S. Yesu Doss Philip and R.Kala ,4-total difference cordial labeling of some of graphs (Submitted to the journal).*